Published by Institute of Physics Publishing for SISSA/ISAS
Received: *March 12, 2003*
Accepted: *May 5, 2003*

Gauge/string duality and scalar glueball mass ratios

Henrique Boschi-Filho and Nelson R. F. Braga

Instituto de Física, Universidade Federal do Rio de Janeiro
Caixa Postal 68528, RJ 21945-970, Brazil
E-mail: boschi@if.ufrj.br, braga@if.ufrj.br

Abstract: It has been shown by Polchinski and Strassler that the scaling of high energy QCD scattering amplitudes can be obtained from string theory. They considered an AdS slice as an approximation for the dual space of a confining gauge theory. Here we use this approximation to estimate in a very simple way the ratios of scalar glueball masses imposing Dirichlet boundary conditions on the string dilaton field. These ratios are in good agreement with the results in the literature. We also find that they do not depend on the size of the slice.

Keywords: Field Theories in Higher Dimensions, AdS-CFT and dS-CFT Correspondence.

JHEP05(2003)009

Glueballs are bound states of gluons as predicted by QCD, presently the main candidate theory to describe the strong interactions. These bound states have not yet been observed but hopefully will appear in near future accelerators. The investigation of such states using QCD in the low energy (strong coupling) regime, in particular looking for their masses, requires a non perturbative approach. In this case lattice calculations produce interesting results (see for instance [1, 2] and references therein).

An alternative approach to strong interactions is based on the idea that they have a description in terms of strings [3, 4]. A remarkable step in this direction was given by Maldacena [5] proposing the equivalence between conformal fields and string theory in anti-de Sitter spacetime (AdS/CFT correspondence) [6, 7, 8]. In particular glueball operators of the conformal gauge theory defined on the AdS boundary are in correspondence with the string dilaton field.

The description of strong interactions based on this correspondence requires the breaking of conformal invariance, which can be done in different ways. Witten [9] proposed a formulation of the correspondence in such a way that the AdS space accommodates a Schwarzschild black hole. This procedure introduces a scale breaking conformal invariance. Then it is possible to obtain the ratio of glueball masses from supergravity approximation to string theory. The corresponding supergravity equations do not allow analytic solutions but the eigenvalues related to the glueball masses can be found using a WKB method [10].

Another possibility to break conformal invariance is to consider a slice (actually two slices sticked together) of the AdS space as in the Randall-Sundrum model [11, 12] which proposes a unification of the weak and Planck scales. Such an AdS slice was used recently by Polchinski and Strassler [13] as an approximation for the string dual space corresponding to strong interactions (see also [14]–[17]). They used this model to reproduce the scaling of high energy glueball scattering amplitudes as predicted by QCD [18, 19]. Following the Polchinski and Strassler proposal we consider an AdS slice assuming that there is still a bulk/boundary correspondence, in particular between scalar glueballs and the dilaton. Imposing boundary conditions the string dilaton field acquires discrete modes. Then it is natural to relate the spectrum of the dilaton field to the masses of the scalar glueballs. Our results are in good agreement with lattice and supergravity calculations (see tables 1 and 2).

4d Glueball	lattice, $N = 3$	AdS-BH	AdS slice
0^{++}	1.61 ± 0.15	1.61 (input)	1.61 (input)
0^{++*}	2.8	2.38	2.64
0^{++**}	-	3.11	3.64
0^{++***}	-	3.82	4.64
0^{++****}	-	4.52	5.63
$0^{++*****}$	-	5.21	6.62

Table 1: Masses of the first few 0^{++} four dimensional glueballs with SU(N) and $N = 3$, in GeV, from lattice QCD [1, 2], from AdS-Schwarzschild black hole supergravity (AdS-BH) [10] and our results from AdS slice normal modes eq. (7).

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3d Glueball	lattice, $N=3$	lattice, $N \to \infty$	AdS-BH	AdS slice
0^{++}	4.329 ± 0.041	4.065 ± 0.055	4.07 (input)	4.07 (input)
0^{++*}	6.52 ± 0.09	6.18 ± 0.13	7.02	7.00
0^{++**}	8.23 ± 0.17	7.99 ± 0.22	9.92	9.88
0^{++***}	-	-	12.80	12.74
0^{++****}	-	-	15.67	15.60
$0^{++*****}$	-	-	18.54	18.45

Table 2: 0^{++} three dimensional glueball masses with $SU(N)$ from lattice QCD [1, 2] (in units of string tension), from AdS-Schwarzschild black hole supergravity (AdS-BH) [10] and our results from AdS slice normal modes, eq. (8).

According to the AdS/CFT correspondence string theory defined on AdS_5 times a transverse space X is dual to the large-N limit of $SU(N)$ conformal gauge theories with extended supersymmetry defined on the four dimensional boundary. The metric for this space can be written as

$$ds^2 = \frac{R^2}{z^2}\Big(dz^2 + (d\vec{x})^2 - dt^2\Big) + R^2 ds_X^2\,, \tag{1}$$

where $z, \vec{x}, t$ are the Poincare coordinates that describe the AdS space with radius R and ds_X^2 corresponds to the metric of a convenient transverse space, as for example S^5.

We consider an AdS slice corresponding to the region $0 \le z \le z_{\rm max}$. According to the AdS/CFT duality, there is a holographic relation between bulk and boundary theories such that low energies correspond to large z-values. Then $z_{\rm max}$ is an infrared cut off for the nonconformal boundary theory. Further we take the dilaton momenta associated with the directions of the space X to be negligible. Then choosing Dirichlet boundary condition the free dilaton field at $z = z_{\rm max}$ can be cast in terms of the Bessel function J_2 into the form [20]

$$\Phi(z,\vec{x},t) = \sum_{p=1}^{\infty} \int \frac{d^3k}{(2\pi)^3}\, \frac{z^2\, J_2(u_p z)}{z_{\rm max}\; w_p(\vec{k})\; J_3(u_p z_{\rm max})} \times \Big\{ \mathbf{a}_p(\vec{k}) e^{-i w_p(\vec{k}) t + i\vec{k}\cdot\vec{x}} + {\rm h.c.} \Big\}\,. \tag{2}$$

Here $w_p(\vec{k}) = \sqrt{u_p^2 + \vec{k}^2}$, h.c. means hermitian conjugate and u_p are the discrete momenta associated with coordinate z, defined by

$$u_p = \frac{\chi_{2\,,p}}{z_{\rm max}}\,, \tag{3}$$

where $J_2(\chi_{2\,,p}) = 0$, and the operators $\mathbf{a}_p\,, \mathbf{a}_p^\dagger$ satisfy canonical commutation relations.

On the boundary ($z = 0$) of the AdS slice we consider composite operators representing glueballs with masses μ_p. As in ref. [13] we associate the size $z_{\rm max}$ of the AdS slice with the mass of the lightest glueball μ_1

$$z_{\rm max} = \frac{C}{\mu_1}\,, \tag{4}$$

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where C is an arbitrary constant related to the choice of $z_{\max}$. From equations (3) and (4) we have

$$u_p = \frac{\chi_{2\,,p}}{C}\mu_1\,. \tag{5}$$

Then the gauge/string correspondence suggests that the glueball masses are proportional to the discrete dilaton modes:

$$\frac{u_p}{\mu_p} = C'\,, \tag{6}$$

where $C' = \chi_{2\,,1}/C$ according to eq. (5).

So the glueball masses are related to the zeros of the Bessel functions by

$$\frac{\mu_p}{\mu_1} = \frac{\chi_{2\,,p}}{\chi_{2\,,1}}\,. \tag{7}$$

This is our main result. It is remarkable that it is independent of the size $z_{\max}$, although the individual masses depend on this cut off.

Using the values of the zeros of the Bessel function, we find the mass ratios for the scalar glueball state $J^{PC} = 0^{++}$ and its excitations. Our results are presented in table 1 together with lattice [1, 2] and AdS-Schwarzschild black hole supergravity [10] calculations. From this table one can see that our simple approximation is in good agreement with these previous calculations.

A similar approach can be used to estimate the glueball masses in QCD_3. In this case we consider AdS_4 and the dilaton fields are expanded in terms of the Bessel function $J_{3/2}$ and the mass ratios for the 3 dimensional "glueballs" are given by

$$\frac{\mu_p}{\mu_1} = \frac{\chi_{3/2\,,p}}{\chi_{3/2\,,1}}\,. \tag{8}$$

Using this relation we obtain the ratio of masses presented in table 2 together with lattice and AdS-Schwarzschild black hole supergravity calculations. The agreement here is also good.

The relations (7), (8) for the ratio of glueball masses were found by us before in [21] using a mapping between bulk and boundary quantum states. Here we have seen that these results do not depend on any particular mapping between quantum states.

In conclusion we have seen that string theory defined in an AdS slice can be applied to estimate the scalar glueball mass ratios in a very simple way. We hope that such an approach can be used to estimate the mass ratios of other strongly interacting states.

Acknowledgments

We would like to thank J. R. T. Mello Neto for important discussions. The authors are partially supported by CNPq, FINEP , CAPES (PROCAD) and FAPERJ - Brazilian research agencies.

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